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COMPUTATIONAL INTELLIGENCE APPROACHES FOR COMPARATIVE ANALYSIS OF EXACT AND ASYMPTOTIC SOLUTIONS IN OSCILLATORY VISCOUS PIPE FLOW

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KEYWORDS

Viscous fluid, transfer function, oscillatory flow, asymptotic formula, pulsating flow, cylindrical pipe, pipe flow

ABSTRACT

This paper presents an analytical and computational study of oscillatory flow of a viscous incompressible fluid in a cylindrical pipe subjected to harmonic pressure oscillations. Exact solutions for the velocity field and the cross-sectional average velocity are derived and used to determine the transfer function, amplitude–frequency, and phase–frequency characteristics of the flow. The effect of oscillation frequency on the ratio between wall shear stress and average flow velocity is investigated. By comparing exact and asymptotic solutions, new asymptotic formulas are developed to improve the efficiency of predictive modeling. The obtained results provide a reliable mathematical framework for data-driven modeling, physics-informed machine learning, and intelligent monitoring of unsteady fluid transport processes in pipeline systems.

Introduction

Oscillatory flows of viscous fluids in cylindrical pipes are encountered in numerous engineering, biomedical, and microfluidic applications. In particular, pneumatic micropumps used in medical and biological devices often operate under conditions of periodically varying flow rates. Such systems are widely employed for diagnostic purposes, targeted drug delivery, and the transportation of biological fluids. Accurate prediction of the hydrodynamic characteristics of oscillatory flows is therefore essential for the design and optimization of these devices [1,2].

The presence of flow pulsations significantly affects the pressure gradient, hydraulic resistance, and wall shear stress. However, the influence of oscillation frequency on frictional losses and flow resistance characteristics remains insufficiently understood [3–5]. A detailed investigation of these effects is important for both biomedical applications and various industrial fluid transport systems. Theoretical studies of oscillatory flows have been conducted by many researchers. In [6], unsteady pulsating flow in an infinitely long circular pipe subjected to a harmonic pressure gradient was investigated, and analytical expressions for the velocity distribution and volumetric flow rate were obtained. Oscillatory flows of incompressible fluids in rectangular channels were studied in [7], where numerical solutions were used to determine the amplitude and phase characteristics of velocity oscillations.

Numerical approaches for modeling isothermal and non-isothermal flows of non-Newtonian fluids with free surfaces were presented in [8]. The study focused on the development of mathematical models and computational methods for fluid flow in planar and axisymmetric channels. The oscillatory motion of viscoelastic fluids has also attracted considerable attention. In [9], the flow of a viscoelastic fluid through a long pipe under the action of an oscillating pressure gradient was analyzed, and the distinctive features of viscoelastic behavior were compared with those of Newtonian fluids. Oscillatory flows of Maxwell and Oldroyd-B fluids were investigated in [10],

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revealing several unique dynamic phenomena absent in Newtonian flows.

Further studies [11–17] examined unsteady oscillatory flows of viscoelastic fluids in planar channels based on the Maxwell model. Analytical expressions for dynamic and frequency-response characteristics were derived, demonstrating that fluid elasticity and acceleration impose important limitations on the applicability of quasi-steady approximations. Electrokinetic oscillatory flows of viscoelastic fluids were investigated in [18], where resonance effects associated with the elastic properties of Maxwell fluids were identified.

Although the aforementioned studies provide extensive information regarding velocity distributions and pressure-driven oscillatory flows, considerably less attention has been devoted to the analysis of wall shear stress and its dynamic relationship with the flow rate. Understanding this relationship is essential for characterizing energy dissipation, hydraulic losses, and transient transport processes in oscillatory fluid systems.

Therefore, the present study investigates the oscillatory flow of a viscous incompressible fluid in a cylindrical pipe for a prescribed harmonic variation of the cross-sectional average velocity. Exact analytical solutions are obtained, and the corresponding transfer function together with the amplitude–frequency and phase–frequency characteristics are derived. The influence of the dimensionless oscillation frequency on the wall shear stress and fluid acceleration is analyzed. Furthermore, a comparison between exact and asymptotic solutions is performed, leading to the development of new asymptotic formulas. These formulas provide an efficient tool for analyzing oscillatory flows and may serve as a basis for advanced computational and data-driven modeling of unsteady viscous and viscoelastic fluid transport processes.

II. Problem statement and solution method

Let us formulate the problem of slow oscillatory flow of a viscous fluid in a cylindrical pipe of infinite length. Let us denote the radius of the pipe by R. Axis $0x$ runs horizontally in the middle of the pipe along the flow, and the axis $0r$ directed perpendicular to the axis $0x$ The differential equation of motion of a viscous incompressible fluid has the following form [19-21]:

$$\rho\frac{\partial \vartheta_x}{\partial t}=-\frac{\partial p}{\partial x}+\eta\left(\frac{\partial^2 \vartheta_x}{\partial r^2}+\frac{1}{r}\frac{\partial \vartheta_x}{\partial r}\right) \quad (1)$$

Where ϑ_x-longitudinal speed; p-pressure; ρ-density; η-dynamic viscosity; t-time.

We assume that the oscillatory flow of a viscous fluid occurs due to the longitudinal velocity averaged over the cross-section of the pipe:

$$<\vartheta_x>=a_{\vartheta_x}\cos\omega t=\mathrm{Re}\,a_{\vartheta_x}e^{i\omega t} \quad (2)$$

Where a_{ϑ_x} - amplitude of longitudinal velocity averaged over the cross-section pipes; ω-oscillation frequency. The no-slip conditions are satisfied at the pipe walls, and the longitudinal velocity is limited at the center. Then the boundary conditions are:

$$\vartheta_x=0 \text{ at } r=R$$
$$\vartheta_x=0 \text{ at } r=R \quad (3)$$

Due to the linearity of equation (1) of longitudinal velocity, pressure, and shear stress on the wall can be written as follows:

$$\vartheta_x(r,t)=\mathrm{Re}\,\vartheta_{x1}(r)e^{i\omega t},\ p(x,t)=\mathrm{Re}\,p_1(x)e^{i\omega t},\tau(t)=\mathrm{Re}\,\tau_1 e^{i\omega t} \quad (4)$$

Substituting these expressions into equation (1), we obtain:

$$\left(\frac{\partial^2 \vartheta_{x1}(r)}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)-\frac{\rho i\omega}{\eta}\vartheta_{x1}(r)=\frac{1}{\eta}\frac{\partial p_1(x)}{\partial x} \quad (5)$$

$\alpha_0=\sqrt{\frac{\omega}{\nu}}R$ - Womersley oscillation number (dimensionless oscillation frequency); ν - kinematic viscosity.

Solving equation (5) taking into account the boundary conditions (3), we have

$$\vartheta_{x1}(r)=\frac{1}{\rho i\omega}\left(-\frac{\partial p_1(x)}{\partial x}\right)\left(1-\frac{J_0\left(i^{3/2}\alpha_0\right)\frac{r}{R}}{J_0\left(i^{3/2}\alpha_0\right)}\right) \quad (6)$$

Using the equation

$$\tau_1=-\eta\frac{\partial \vartheta_{x1}(r)}{\partial r}\Big|_{r=R} \quad (7)$$

we find the shear stress on the wall

$$\tau_1=-R\left(-\frac{\partial P}{\partial x}\right)\frac{1}{i\alpha_0^2}\left(\frac{i^{3/2}\alpha_0 J_1\left(i^{3/2}\alpha_0\right)}{J_0\left(i^{3/2}\alpha_0\right)}\right) \quad (8)$$

Multiplying both sides of equation (6) $2\pi r$ and integrating them from zero to R by variable r, Let's find formulas for liquid flow rate

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$$Q_1 = \pi R^2 \left[\frac{1}{\rho i\omega} \left(-\frac{\partial p_1(x)}{\partial x} \right) \left(1 - \frac{2J_1\left(i^{3/2}\alpha_0\right)}{\left(i^{3/2}\alpha_0\right) J_0\left(i^{3/2}\alpha_0\right)} \right) \right] \quad (9)$$

and taking into account in formula (9) $Q_1 = \pi R^2 < \vartheta_{x1} >$, we find the longitudinal velocity averaged over the cross-section of the pipe

$$< \vartheta_{x1} >= \frac{1}{\rho i\omega} \left(-\frac{\partial p_1(x)}{\partial x} \right) \left(1 - \frac{2J_1\left(i^{3/2}\alpha_0\right)}{\left(i^{3/2}\alpha_0\right) J_0\left(i^{3/2}\alpha_0\right)} \right) \quad (10)$$

Here $\rho i\omega$ can be written as

$$\rho i\omega = i\frac{\omega}{\nu} R^2 \cdot \frac{\eta}{R^2} = i\alpha_0^2 \frac{\eta}{R^2} \quad (11)$$

Then formula (10) takes the form:

$$< \vartheta_{x1} >= \frac{R}{4\eta} \tau_1 \cdot \frac{4J_2(i^{\frac{3}{2}}\alpha_0)}{\left(i^{\frac{3}{2}}\alpha_0\right) J_1(i^{\frac{3}{2}}\alpha_0)} \quad (12)$$

Transfer function $W_{\tau_1,\vartheta_{x1}}\left(i\omega\right)$ we define using formulas (11) as:

$$W_{\tau_1,\vartheta_{x1}}\left(i\omega\right) = \frac{R}{4\eta} \frac{\tau_1\left(i\omega\right)}{< \vartheta_{x1}\left(i\omega\right) >} \quad (13)$$

From equation (11) we obtain

$$W_{\tau_1,\vartheta_{x1}}\left(i\omega\right) = \frac{R}{4\eta} \frac{\tau_1\left(i\omega\right)}{< \vartheta_{x1} >} = \frac{\left(i^{\frac{3}{2}}\alpha_0\right) J_1\left(i^{\frac{3}{2}}\alpha_0\right)}{4J_2(i^{\frac{3}{2}}\alpha_0)} \quad (14)$$

The transfer function (13) is called the amplitude-phase frequency response (APFR). These functions allow us to determine the time dependence of the shear stress on the pipe wall for a given law of change in the longitudinal velocity averaged over the flow cross-section. As is well known, in most cases, when solving transient problems, shear stresses on the wall obtained under quasi-steady fluid flow conditions are used. In real cases, such assumptions are valid when the distribution of local velocities over the flow cross-section has a parabolic distribution law. In this case, the shear stress on the pipe wall oscillates in phase with the oscillation of the averaged longitudinal velocity over the flow cross-section.

Then size $\tau_{o,kc}$ can be calculated using the formula:

$$\tau_{o,kc} = \frac{4\eta}{R} < \vartheta_{x1} > \quad (15)$$

And instead of a quasi-stationary flow of shear stress on the wall $\tau_{o,kc}$, you can accept:

$$\tau_{o,kc} = \tau_{нс} \quad (16)$$

Thus, relation (16) makes it possible to replace the quantity $\tau_{нс}$ for meaning $\tau_{o,kc}$, only under the condition that the actual distribution of local velocities over the flow cross-section differs little from the quasi-stationary one. However, in many cases in an unsteady flow, the law of distribution of local velocities differs significantly from the quasi-stationary one. In most studies [8-15,18-21] it was shown that in oscillatory laminar flow in a cylindrical pipe, the change in local velocities in the near-pier layers outpaces the change in local velocities in time than in the central layers. In an oscillatory flow, due to the change in the law of distribution of local velocities over the channel cross-section, the values □$_{нс}$ in reality differs significantly from . The linear model of unsteady flow provides the most complete understanding of the dependence on □$_{окс}$□$_{нс}$ $< \vartheta_{x1} >$ can be obtained using the transfer function (13).

III. Calculation results and analysis

Using the transfer function (13), we investigate the dependence of the shear stress on the longitudinal velocity averaged over the cross-section of the pipe, when the law of change in the longitudinal velocity is given in the form:

$$< \vartheta_{x1} >= a_{\vartheta_{x1}} \cos\omega t \quad (17)$$

Where $a_{\vartheta_{x1}}$ -the amplitude of the longitudinal velocity averaged over the cross-section of the pipe. Due to the linearity of equations (16), the change in the shear stress will also be harmonic, but in the general case, shifted in phase with respect to $< \vartheta_{x1} >$. Then, the change in shear stress is determined as follows:

$$\tau_1 = a_{\tau_1} \cos(\omega t + \varphi_{\tau_1}) \quad (18)$$

Where a_{τ_1} - amplitude of shear stress; φ_{τ_1} - phase shift between the magnitude τ_1 and $< \vartheta_{x1} >$. Using the ratio

$$\cos(\omega t+\varphi_{\tau_1})=\cos\omega t\cos\varphi_{\tau_1}-\sin\omega t\sin\varphi_{\tau_1} \quad (19)$$

And considering that

$$\frac{\partial<\vartheta_{x1}>}{\partial t}=-a_{\vartheta_{x1}}\omega\sin\omega t \quad (20)$$

Equation (18) will take the form

$$\tau_1=(\frac{a_{\tau_1}}{a_{\vartheta_{x1}}}\cos\varphi_{\tau_1})<\vartheta_{x1}>+(\frac{a_{\tau_1}}{a_{\vartheta_{x1}}}\sin\varphi_{\tau_1})\frac{1}{\omega}\frac{\partial<\vartheta_{x1}>}{\partial t}\ (\quad 21)$$

Quantities $(\frac{a_{\tau_1}}{a_{\vartheta_{x1}}}\cos\varphi_{\tau_1})$ and $\frac{a_{\tau_1}}{a_{\vartheta_{x1}}}\sin\varphi_{\tau_1}$ are the real and imaginary parts of the transfer function (13), respectively, therefore from (16) we obtain:

$$W_{\tau_1,\vartheta_{x1}}=\frac{1}{4}(\frac{\left(i^{\frac{3}{2}}\alpha_0\right)J_1\left(i^{\frac{3}{2}}\alpha_0\right)}{J_2(i^{\frac{3}{2}}\alpha_0)})=\chi+\beta i \quad (22)$$

Here $\chi=(\frac{a_{\tau_1}}{a_{\vartheta_{x1}}}\cos\varphi_{\tau_1})$ valid and $\beta=\frac{a_{\tau_1}}{a_{\vartheta_{x1}}}\sin\varphi_{\tau_1}$ -imaginary part of the transfer function (13).

Taking into account (22) in (21), we obtain:

$$\frac{R}{4\eta}\frac{\tau_1}{<\vartheta_{x1}>}=W_{\tau_1,\vartheta_{x1}}=\chi+\beta\frac{1}{\omega}K_{н} \quad (23)$$

Here $K_{н}=\frac{\partial<\vartheta_{x1}>}{<\vartheta_{x1}>\partial t}$ - the parameter determines the relative acceleration of the liquid, χ and β -dimensionless quantities, t dimensional quantities, so it needs to be converted to a dimensionless form using the transformation:

$$t=\frac{R^2\rho}{8\eta}t^* \quad (24)$$

Taking into account (15) and (24) on (23), we obtain the calculation formula:

$$\frac{\tau_{нс}}{\tau_{0кс}}=\chi+\frac{8\beta}{\alpha_0^2}K^*_{н} \quad (25)$$

Here $K^*_{н}=\frac{\partial<\vartheta_{x1}>}{<\vartheta_{x1}>\partial t^*}$, $\tau_{0кс}=\frac{4\eta}{R}<\vartheta_{x1}>$ and $\tau_1=\tau_{нс}$ χ and β are determined using formula (22).

For calculations, instead of Bessel functions, one can use Thomson functions, taking into account that:

$$J_n(i^{3/2}\alpha_0)=ber_n(\alpha_0)+ibei_n(\alpha_0) \quad (26)$$

then the amplitude-phase frequency characteristic (22) will take the form

$$W_{\tau_1,\vartheta_{x1}}(i\omega)=\frac{\left(i^{\frac{3}{2}}\alpha_0\right)(ber_1(\alpha_0)+ibei_1(\alpha_0))}{4(ber_2(\alpha_0)+ibei_2(\alpha_0))} \quad (27)$$

Where $ber_1(\alpha_0), bei_1(\alpha_0), ber_2(\alpha_0), bei_2(\alpha_0)$ - Thomson function for a given value α_0 can be calculated using special tables [19-23].

Having isolated the real and imaginary parts of the phase-frequency response (24), we obtain:

$$\chi=\mathrm{Re}[W_{\tau_1,\vartheta_{x1}}(i\omega)]=\frac{a_{\tau_1}}{a_{\vartheta_1}}\cos\varphi_{\tau_1}=\mathrm{Re}[\frac{\left(i^{\frac{3}{2}}\alpha_0\right)(ber_1(\alpha_0)+ibei_1(\alpha_0))}{4(ber_2(\alpha_0)+ibei_2(\alpha_0))}]= \quad (28)$$

$$\frac{\alpha_0}{4\sqrt{2}}\frac{(ber_1\alpha_0(bei_2\alpha_0-ber_2\alpha_0)-bei_1\alpha_0(bei_2\alpha_0+ber_2\alpha_0))}{(ber_2^2\alpha_0+bei_2^2\alpha_0)}$$

$$\beta=\mathrm{Im}[W_{\tau_1,\vartheta_{x1}}(i\omega)]=\frac{a_{\tau_1}}{a_{\vartheta_1}}\sin\varphi_{\tau_1}=\mathrm{Im}[\frac{\left(i^{\frac{3}{2}}\alpha_0\right)(ber_1(\alpha_0)+ibei_1(\alpha_0))}{4(ber_2(\alpha_0)+ibei_2(\alpha_0))}]= \quad (29)$$

$$\frac{\alpha_0}{4\sqrt{2}}\frac{(ber_1\alpha_0(bei_2\alpha_0+ber_2\alpha_0)+bei_1\alpha_0(bei_2\alpha_0-ber_2\alpha_0))}{(ber_2^2\alpha_0+bei_2^2\alpha_0)}$$

Despite the fact that the Thomson functions are tabulated, calculations using formulas (28) and (29) lead to rather cumbersome computations. Furthermore, they cannot be used for oscillatory flow of a viscoelastic fluid in a cylindrical pipe. Therefore, if we assume that for large values of , the dimensionless oscillation frequencies, i.e., $\left|i^{3/2}\alpha_0\right|>1$ We will expand the Bessel functions contained in (19) into asymptotic series, taking into account the formula [19]:

$$i^{-n}J_n(ix)=\frac{e^x}{\sqrt{2\pi x}}[1-\frac{4n^2-1}{1!(8x)}+\frac{(4n^2-1)(4n^2-3^2)}{2!(8x)^2}-...]. \quad (30)$$

Substituting instead x its meaning $x=\sqrt{i}\alpha_0$ and keeping in the series under consideration only three members with rounding of the coefficients when X , one can find an approximate expression (19) in the form

$$W_{\tau_1,\vartheta_{x1}}=\frac{1}{4}(\frac{(x)J_1(ix))}{J_2(ix))})=\frac{(x)}{4}(1+\frac{12}{1!(8x)}+\frac{15\cdot16}{2!(8x)^2}\cdots) \quad (31)$$

Considering that $x=\sqrt{i}\alpha_0$ in (31), we

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obtain

$$W_{\tau_1,\vartheta_{x1}} = \frac{1}{4}\left(\frac{\left(i\sqrt{i}\alpha_0\right)J_1\left(i\sqrt{i}\alpha_0\right)}{J_2(i\sqrt{i}\alpha_0)}\right) = \frac{\left(\sqrt{i}\alpha_0\right)}{4}\left(1+\frac{12}{1!(8\sqrt{i}\alpha_0)}+\frac{15\cdot 16}{2!(8\sqrt{i}\alpha_0)^2}\cdots\right) = \chi+\beta i \quad (32)$$

For oscillatory flow, from formula (32), having isolated its real and imaginary parts, we obtain:

$$\chi = \mathrm{Re}[W_{\tau_1,\vartheta_{x1}}(i\omega)] = \frac{\alpha_0}{4\sqrt{2}}+\frac{3}{8}+\frac{15}{32\sqrt{2}\alpha_0}+\ldots \quad (33)$$

$$\beta = \mathrm{Im}[W_{\tau_1,\vartheta_{x1}}(i\omega)] = \frac{\alpha_0}{4\sqrt{2}}-\frac{15}{32\sqrt{2}\alpha_0}+\cdots \quad (34)$$

Based on formula (26), taking into account formulas (29), (30) and (34), graphs were constructed in Fig. 1, showing the change in the relative shear stress in a non-stationary flow depending on the dimensionless oscillation frequency.

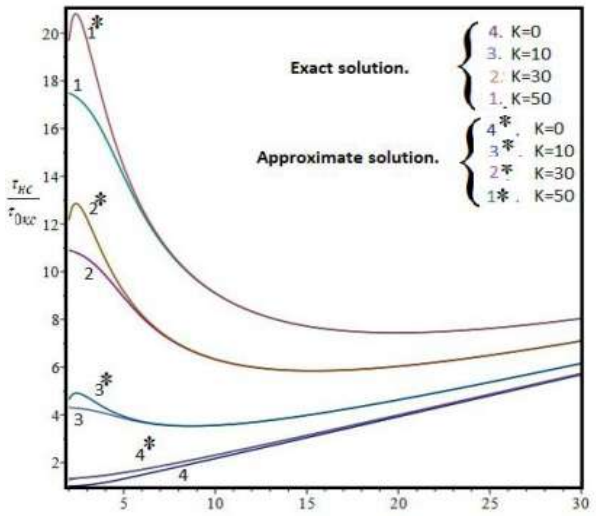


Fig. 1

From these graphs it is evident that the change in the relative shear stress in a non-stationary flow depending on the dimensionless oscillation frequency, at $\alpha_0 = 2$, differ by no more than 2% from their exact values, and when $\alpha_0 > 2$ these differences approach zero. Thus, in this case, the approximate asymptotic formulas (30) can be applied with high accuracy, starting from $\alpha_0 = 2$ to study the oscillatory flow of viscous and viscoelastic fluid in a cylindrical pipe.

Conclusion

The problems of oscillatory flow of a viscous incompressible fluid in a cylindrical pipe with a given harmonic oscillation are solved average velocity across the pipe cross-section. The transfer function and amplitude-phase frequency characteristics were determined. Using these functions, the influence of oscillation frequency and acceleration on the ratio of shear stress to the cross-sectional average velocity was determined. Asymptotic formulas for determining the transfer function for the flow of a viscous fluid in a non-stationary flow at lower and higher values of the oscillation frequency were found. By comparing the exact solution of the problem with the asymptotic solution, new asymptotic formulas are proposed that are of great importance in the study of the oscillatory flow of a viscoelastic fluid in a cylindrical pipe.

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